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What about other values of *x*? Choose another value for *x*, such as 2, and prepare several canisters with a value of 2. Using these canisters repeat the procedure starting with $x = x$ on page 10. Again the balance stays in equilibrium; therefore, the equation is true when $x = 2$. No matter how many times you repeat this process, using different values for the canisters, each time the balance will stay in a state of equilibrium. Thus, this process shows if an equation is true for any value of *x*, the equation is an identity.

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ALGEBRA BALANCE

LER 7545

By Dr. George Kung and Ken Vicchiollo

Getting Started

The Algebra Balance is a unique tool for helping students make sense of algebraic concepts. Now students can show –1 is less than 0 as they participate in a hands-on, interactive learning experience. Allow students ample time to experiment with the balance. Give them the opportunity to learn how to manipulate the chips and canisters in order to keep the balance in equilibrium. Remind students to allow the balance and pans to steady before recording any information.

Manipulatives provide an experimental basis for the development of abstract ideas. Using the Algebra Balance, students can physically represent and solve equations. As students become more proficient in using the balance, they can investigate solving a system of linear equations, using either one balance or two balances simultaneously.

The following pieces are included with your Algebra Balance:

- 4 canisters
- 36 chips
- 2 short hooks
- 2 long hooks
- 2 yellow pans
- 2 red pans
- 2 short hangers
- 2 long hangers
- 1 main balance
- 1 base
- 2 arms

⚠ WARNING:
CHOKING HAZARD - Small parts.
Not for children under 3 years.

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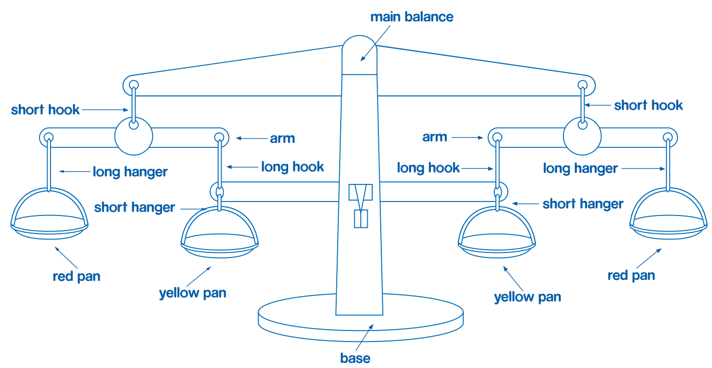
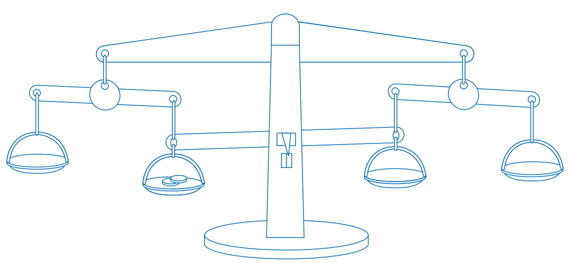
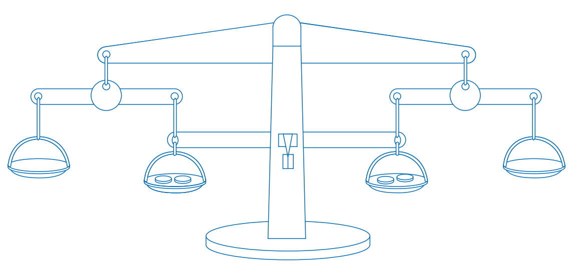


Using the Algebra Balance

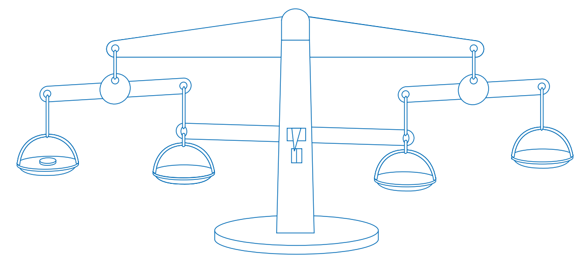
Weights, consisting of chips and canisters, are used to model positive and negative numbers and equations. One empty canister with the lid weighs the same as 1 chip. Chips placed in canisters create the variables, or unknowns.
The inner two pans (yellow) and the beam between them operate like a normal swing balance. Putting chips in one of the inner pans, as shown below, makes that side heavier and the beam tilts down toward that side. The indicator points to the lighter side.



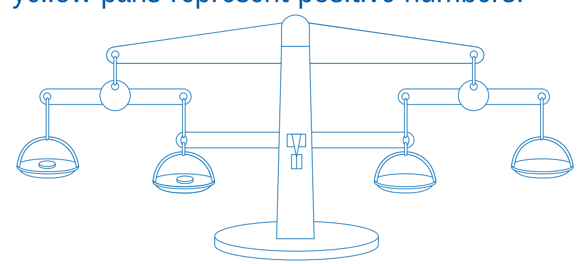
Putting the same number of chips in each of the inner pans brings the balance into equilibrium. For example, in the illustration shown below, 2 chips have been placed in each inner pan. Notice all beams are level and the indicator is in the center.



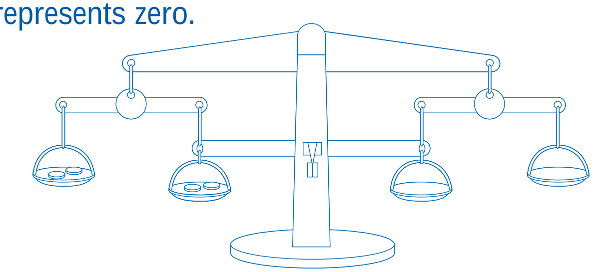
The outer two pans (red) are a unique feature of the Algebra Balance. Put 1 chip in the left-hand outer pan. Focusing for a moment only on the two inner pans and their connecting beam, notice, as shown below, that the left-hand yellow pan goes up, indicating it is lighter than the other yellow pan. This means two things. The right side of the balance is heavier and represents 0 because it's pans are empty. The left side of the balance has a negative value.



Now place a chip in the left-hand yellow pan. Watch as the balance returns to equilibrium – showing that the two chips represent zero. Assigning the original chip the value of n, the balance can be described symbolically as $n + 1 = 0 + 0$. The solution to this equation, $n = -1$, shows that chips placed in the outer red pans of the balance represent negative numbers, whereas the chips placed in the inner yellow pans represent positive numbers.



Placing 2 chips (or 3, or 4, or any number) in each pan on the left side always keeps the balance in equilibrium. Such a "balanced" situation represents zero.



$-2 + 2 = 0 + 0$

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You can also represent zero by removing 1 chip from each pan on the same side of the balance. In the above example, this would result in -1 + 1 = 0 + 0.

The Zero Principle

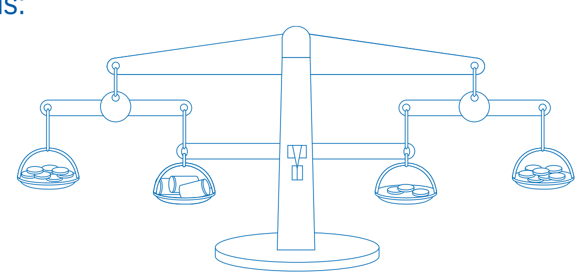
The placement or removal of a pair of equal weights from two pans on the same side of the balance does not affect the equilibrium of the balance. This action models the Zero Principle and is represented algebraically as a + -a + 0.

Any quantity of equal weights placed in the two pans on the same side of the balance represents zero. The two equal quantities are called additive inverses or opposites.

Exploring Integers

The Zero Principle can be used to explore adding and subtracting integers. For example, find the value of -7 + 3.

Begin by modeling -7 + 3 on each side of the balance. On the left side of the balance use chips for one number and empty covered canisters for the other number. On the right side of the balance, use chips for both numbers. The balance should be in equilibrium and look like this:



The left side of the balance represents the problem and will remain untouched. Apply the Zero Principle to the right side of the balance by repeatedly removing 1 chip from each pan until it is no longer possible, that is, until one pan is empty. The balance should remain in equilibrium and show -7 + 3 = -4.

Here is a recording of each step you took:

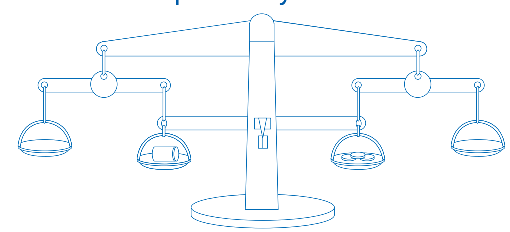
$$\begin{aligned} -7 + 3 &= 3 + (-7) \\ &= 2 + (-6) \\ &= 1 + (-5) \\ &= 0 + (-4) \\ &= -4 \end{aligned}$$

Modeling Equivalent Linear Equations

Before using the balance to solve a linear equation, it is helpful to experiment with a known equality to create and model equivalent equalities. This is done by adding and removing chips and canisters in such a way that the balance always remains in equilibrium.

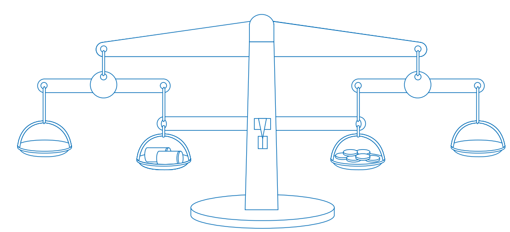
To begin, choose a value for the variable. For example, let x = 3. Prepare several canisters with a value of 3. Since each canister with its lid is equal to the weight of 1 chip, put 2 chips in each canister.

Place 1 canister in the left-hand inner pan (yellow) and 3 chips in the right-hand inner pan (yellow). The balance should be in equilibrium. This is the simplest way to show x = 3.



x = 3

To create the first equivalent equality, double the amount in each pan by adding 1 canister to the left-hand inner pan and 3 chips in the right-hand inner pan. The balance now contains 2 canisters and 6 chips and remains in equilibrium. This doubling action demonstrates the Multiplication Property of Equality: If both members of an equality are multiplied by the same number, the equality is retained. Symbolically, if a = b, then a x c = b x c.



2x = 6

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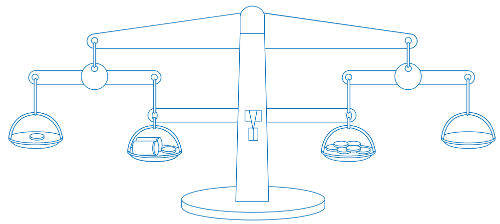
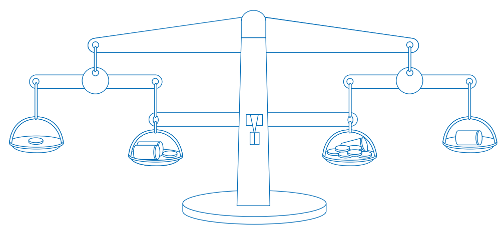
Add a chip to each pan on the left side of the balance. The chip in the red pan represents –1; the chip in the yellow pan, +1. Again the balance remains in equilibrium and the equation it shows is equivalent to $x = 3$. The action of adding the same number of chips to the same side of the balance illustrates the Zero Principle.



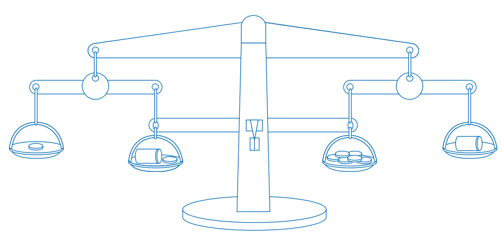
$-1 + 1 + 2x = 6$

Apply the Zero Principle again. Add 1 canister to each pan on the right side of the balance. The canister in the yellow pan represents +x; the one in the red pan, -x. Is the balance in equilibrium? Is the new equation equivalent to $x = 3$? Does your balance match the following illustration?



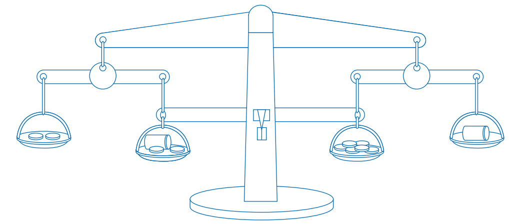
$-1 + 1 + 2x = 6 + x - x$

Remove 1 canister from each of the inner pans to create another equivalency. This action demonstrates the Subtraction Property of Equality: If the same number is subtracted from both members of an equality, the equality is retained. Symbolically, if $a = b$, then $a – c = b – c$.



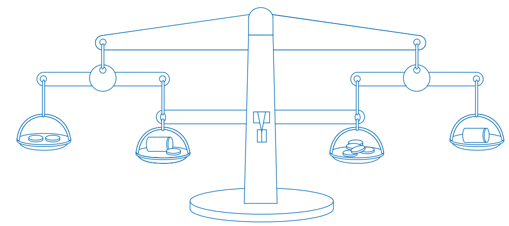
$-1 + 1 + x = 6 - x$

Change the equality once more. Add 1 chip to each of the pans on the left (another application of the Zero Principle) to produce $-2 + 2 + x = 6 – x$.



$-2 + 2 + x = 6 - x$

Finally remove 1 chip from each of the inner pans. Is your balance in equilibrium? Does the balance show $-2 + 1 + x = 5 – x$?



$-2 + 1 + x = 5 - x$

There are many equivalent equations to $x = 3$ that are different from the ones shown in the above sequence. For example, instead of doubling the chips and canisters, as described on page 6, you might start with $x = 3$ and add 2 chips to each of the red pans. This results in an equality since the balance remains in equilibrium. The action represents adding –2 to both sides of the equation, thus, demonstrating the Addition Property of Equality: If $a = b$, then $a + c = b + c$.

Try creating your own sequence of equivalent equations. Try starting with a different equality such as $x = 4$ or $x + 1 = 5$.

Solving Linear Equations

The procedure for solving a linear equation on the Algebra Balance is the same one used to build equivalent linear equations. Using the Zero Principle, Addition/Subtraction Principle of Equality, and the Multiplication Property of Equality, the canisters and chips are manipulated in such a way that the balance stays in equilibrium.

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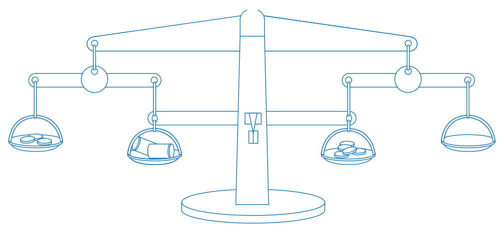
The difference, however, is that solving an equation requires a partnership. One partner (the teacher, a student, or a team) selects an equation whose solution he or she knows and fills the canisters with the correct amount of chips. This is done in secret. The other partner (who must solve the equation using the balance) is given the equation, some chips, and the filled canisters.

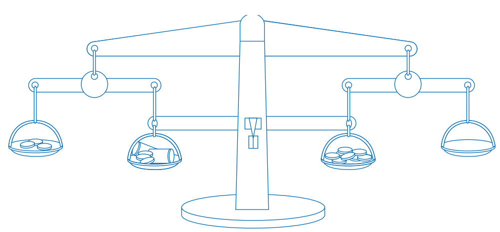
To illustrate, pretend the following. You have been asked to solve the equation 2x – 3 = 5 using the balance. You have been given some chips and several covered canisters properly filled with an amount of chips unknown to you.

First, set up the balance as shown below. Place 3 chips in the left-hand outer pan (red) and 2 canisters in the left-hand inner pan (yellow). On the other side of the balance, place 5 chips in the yellow pan.



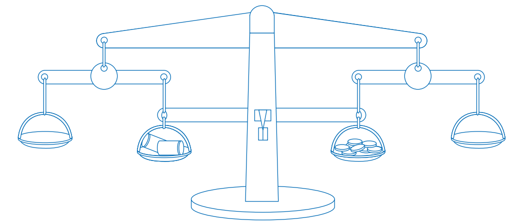
-3 + 2x = 5

To solve the equation, you want to get all the canisters on one side of the balance and all the chips on the other, that is, remove the 3 chips on the left side of the balance. Simply removing them is not an option since the balance will not remain in equilibrium. Instead, add 3 chips to each yellow pan (an application of the Addition Property.)



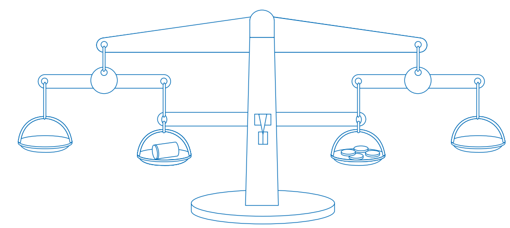
-3 + 3 + 2x = 8

Next, remove 3 chips from each of the pans on the left side of the balance (the Zero Property). Now, the variables are on one side of the balance and the constant terms are on the other. The balance is in equilibrium and shows 2x = 8.



2x = 8

The next step is to have only 1 canister remaining on the left side of the balance. To do this, remove half the contents of each pan, that is 1 canister from one and 4 chips from the other. In effect, you have applied the Multiplication Property of Equality; that is, you have multiplied each member of the equation by $\frac{1}{2}$.



x = 4

The equation is solved. To verify the solution, open the canister and count the number of chips. The 3 chips inside plus the canister and its lid equal 4.

When appropriate, each step performed on the balance should be recorded on paper.

Here is a recording of the steps you just performed:

$$-3 + 2x = 5$$
$$-3 + 3 + 2x = 5 + 3$$
$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

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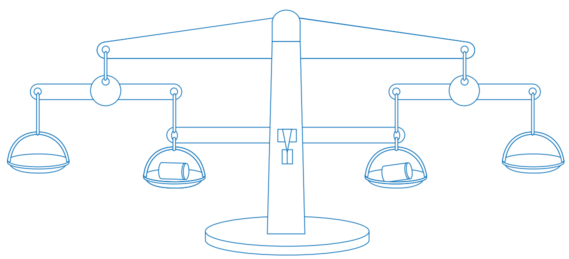
Modeling an Identity

The Algebra Balance can also be used to construct identities. The process is similar to modeling equivalent equations. An identity in one variable, *x*, is an equation such that the equality holds for all values of *x*. In the process of building a linear equation on the Algebra Balance, the value of *x* was used in the first step. If, however, the value of *x* is not used in the construction, the resulting equation is an identity. This translates into always placing or removing 2 canisters on the Algebra Balance. In this way the canister, representing *x*, is never used to achieve any equilibrium.

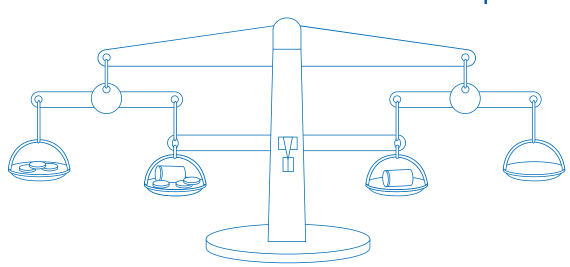
To begin the modeling process, choose a value for the variable. For example, let x = 4. Prepare several canisters with a value of 4. Since each canister with its lid is equal to the weight of 1 chip, put 3 chips in each canister.

Place 1 canister in the left-hand inner pan, (yellow) and 1 canister in the right-hand inner pan (yellow). The balance is in equilibrium.



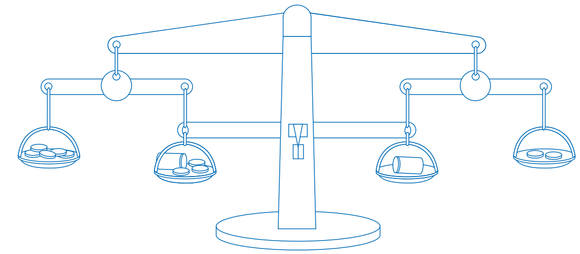
x = x

Now apply the Zero Principle by adding 3 chips to each pan on the left side of the balance. The balance remains in equilibrium.



-3 + x + 3 = x

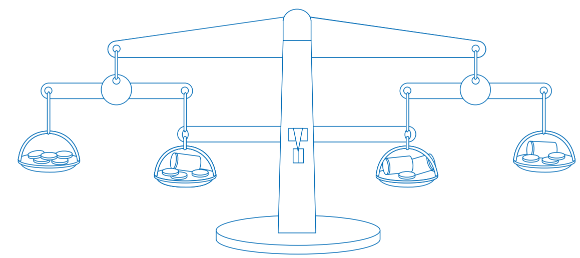
Apply the Addition Property of Equality by adding 2 chips to each outer pan of the balance. This identity can be recorded as -5 + x + 3 = x – 2.



-5 + x + 3 = x - 2

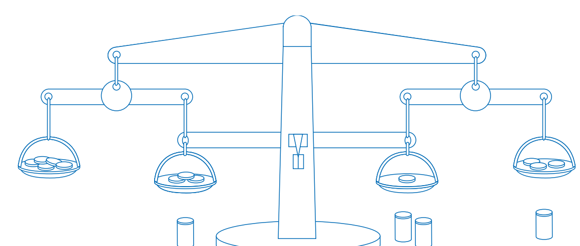
The process of applying the Zero Principle, Addition Property of Equality, and Multiplication Property of Equality can be repeated as many times as you wish.

Assume you arrive at the following arrangement.



-5 + x + 3 = 2x + 1 - x - 3

To show that this is an identity, remove each canister (the variable x) from the balance. Place each one under the pan it was removed from. The balance remains in equilibrium and shows that the equation is true when x = 0. The equation is also true when x = 4 (the original value of the canisters).



-5 + 3 = 1 - 3